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# The impact of risk-averse operation on the likelihood of extreme events in a simple model of infrastructure

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A simple dynamic model of agent operation of an infrastructure system is presented. This system evolves over a long time scale by a daily increase in consumer demand that raises the overall load on the system and an engineering response to failures that involves upgrading of the components. The system is controlled by adjusting the upgrading rate of the components and the replacement time of the components. Two agents operate the system. Their behavior is characterized by their risk-averse and risk-taking attitudes while operating the system, their response to large events, and the effect of learning time on adapting to new conditions. A risk-averse operation causes a reduction in the frequency of failures and in the number of failures per unit time. However, risk aversion brings an increase in the probability of extreme events. © 2009 American Institute of Physics.

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**Many large networked systems, such as those found in critical infrastructures, exhibit characteristics common to near critical complex systems. These characteristics include long time correlations and heavy tails. The interaction of human dynamics with the system is obviously crucial to their operation and in fact is one of the main mechanisms by which they seem to remain near their operational limits (critical point). To explore the impact of different types of behavior on the systems dynamics we have implemented agents coupled to a simple model of a complex infrastructure system to control various aspects of the systems operation and upgrade. We have found that, perhaps contrary to intuition, the system has an increase in large costly failures (a heavier tail) when the agents are risk averse in their operations despite having an overall decrease in the number of failures. With risk taking agents the opposite is true. This has important implications for operational planning as well as modeling complex systems with human dynamics included.**

## I. INTRODUCTION

Infrastructure systems suffer from rare nonperiodic large-scale breakdowns that lead to large economic and other losses by the community and sometimes can cause personal injuries and even loss of lives. The initiating causes of these events are very diverse, ranging from incidents caused by weather, to malfunction of system components and to willful acts. Whether intentional or not, these events can threaten national security. Some examples of such extreme events are the August 14, 2003 blackout in Northeastern America, the consequences of the Katrina hurricane in the New Orleans area, etc.

Today's infrastructure systems are complex technological systems for which such extreme events are "normal accidents." As Perrow<sup>1</sup> indicates, such normal accidents are

characteristic of these systems and it is not possible to eliminate them. Additionally, these extreme events tend to generate a risk-averse attitude in the people managing and operating infrastructure systems. This change in attitude, in turn, modifies the probabilities of occurrence of such events.

Some negative consequences of risk-averse operation on complex systems have been explored by Bhatt *et al.*,<sup>2</sup> who used a model for the propagation of the failures inspired by how forest fires spread.<sup>3</sup> Altmann *et al.*,<sup>4</sup> using a model of human reactions to river floods, showed that the commonly employed method of fighting extreme events by changing protection barriers in reaction to them is generally less efficient than the use of constant barriers to contain them. In this paper, we use a simple model of infrastructures to further explore some of the consequences of such changes in operational attitudes to extreme events. We explore a range of behavior by system operators, varying from risk-taking to risk-averse operation.

We use a very simple model of the cascading process that may lead to extreme events. It is based on the CASCADE model.<sup>5-7</sup> The CASCADE model is a probabilistic model of load-dependent cascading failure, which was inspired by fiber bundle models.<sup>8,9</sup> We assume a system with many components, which is under stress. This stress creates a certain distribution of loads among the components. When a component fails, load is transferred to some of the other components. However, in trying to model the behavior of infrastructure systems like the power grid, instead of using conservation of load on the components it was found to be more appropriate to use a constant load transfer, which leads to a power law distribution of the size of the events. Here we measure the size of an event by the number of failed components.

Here, we extend the CASCADE model to include a dynamic component that evolves in a manner analogous to the

OPA model,<sup>10</sup> which we use to model the behavior of the evolving power grid. Time evolution occurs at two resolutions or scales. Over a short time scale, which is on the order of minutes, small and large cascades can develop. The long time scale corresponds to the evolution of the system over years. The dynamic evolution over the long time scale is governed by a daily increase in consumer demand that raises the overall load on the system and a concurrent engineering response to failures that involves component replacement and upgrades. These components may fail by either overload or random failure. They also have a characteristic lifetime; therefore, timely replacement of the components decreases the probability of failures.

In such a dynamical model the system reaches a steady state in which failures of all sizes may occur at different times. The system is governed by two parameters: the upgrade rate of the components  $\mu$  and the replacement time of the components  $\tau_R$ . Components are replaced periodically as they age by one of the agents who decides when the optimal time to do so is. There is a maximum replacement time, which is set by the average failing time for the component. Additionally, components are of course replaced when they actually fail. Two agents control these parameters and learn how to operate the system in order to maximize a given utility function. Each agent's utility function has a well-defined economic term, which is the profit of selling the services minus the expenses of maintaining and upgrading the system. Each function also includes a penalty for failures, which is not only monetary and in general is proportional to the failure size.

It is through the penalty for failures that we can introduce differing attitudes on the part of the agents. Using basic ideas from prospect theory,<sup>11,12</sup> we can color this penalty and make the attitude of the agents risk averse or risk taking. By changing the agents' attitudes in such a manner, we can evaluate the impact of the changes on the operation of the system. In particular, we are interested on how the probability of extreme events is affected by such changes in the attitude of the agents.

The rest of the paper is organized as follows. In Sec. II, we describe the basic ideas behind the CASCADE model and its extension into a dynamic model. In Sec. III, we discuss the parameters controlling the operation of the system and the agents, which are responsible for the decision-making process leading to their determination. The numerical results of the model are presented in Secs. IV and V. In the former, we discuss the operation of the system with a fixed attitude on part of the agents. In the latter, we discuss the effect of changing the attitude of the agents in response to extreme events. Finally, the conclusions are given in Sec. VI.

## II. A SIMPLE MODEL FOR THE INFRASTRUCTURE SYSTEM

Let us consider a system with  $N$  components. In the spirit of the CASCADE model,<sup>5</sup> we assume that each component  $i$  in the system has a load  $L_i$ . The loads are distributed uniformly between a minimum load  $L_{\min}$  and a maximum load  $L_{\max}$ . The CASCADE model assumes that (1) compo-

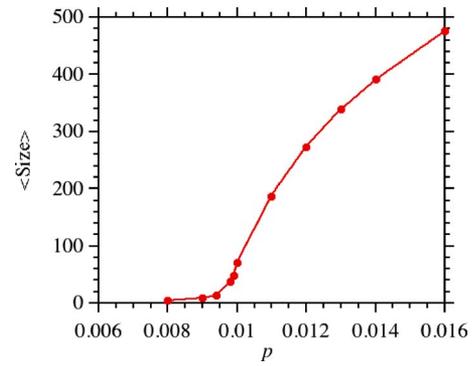


FIG. 1. (Color online) The mean cascade size as a function of  $p$ .

nents fail when their load exceeds a prescribed value  $L^{\text{fail}}$ , and (2) when a component fails, a fixed amount of load  $P$  is transferred to  $k < N$  randomly chosen components.<sup>6,7</sup> In this new version of the model, the components are also characterized by an average lifetime  $T_f$ . This allows for another mechanism for failure. We allow the components to age due to operational stress, which can lead to failure.

The survival function of a component when aging is taken into account is given by<sup>13</sup>

$$S(T) = \exp \left[ - \frac{1}{1 + \beta} \left( \frac{T}{T_f} \right)^{\beta+1} \right], \quad (1)$$

where  $\beta > -1$ . In what follows, we only consider the case of  $\beta = 1$ .

Once one or more components fail, we apply the rules of the CASCADE model listed above, so a constant load  $P$  is transferred to  $k$  components. If any of these components then has as a result a load greater than  $L^{\text{fail}}$ , this component fails in its turn. Thus, the failure of one or more components can possibly lead to a cascading process. The process continues until no more components fail. In what follows we take  $L^{\text{fail}} = L_{\max}$ .

To reduce failures, components are replaced periodically. The replacement time of the components is  $\tau_R$ . The ratio of this replacement time to the failure time  $T_f$  affects the probability of starting a cascade.

In the CASCADE model, all loads can be normalized in the following way:

$$l_i = \frac{L_i - L_{\min}}{L_{\max} - L_{\min}}, \quad p = \frac{P}{L_{\max} - L_{\min}}. \quad (2)$$

After such a normalization, we are left with two independent parameters: the normalized load transfer  $p$  and the ratio of the replacement time to the failure time,  $\tau_R/T_f$ .

The mean size of the cascades increases with the value of  $p$ . There is a critical value of  $p$ ,  $p_c = 1/k$ , after which the cascade size increases sharply. This is shown in Fig. 1, where we have plotted the mean cascade size as a function of  $p$ . The figure is the result of calculations for a system with  $N = 1000$ ,  $k = 100$ , and  $\tau_R/T_f = 0.008$ , so the frequency of cascades in the subcritical regime is low.

One interesting aspect of the CASCADE model is that the distribution of the cascade sizes can be calculated

analytically.<sup>5</sup> At the critical point, the distribution has a power tail with a decaying exponent equal to 1.5.

We have generalized the cascade model by introducing a process involving two time scales. On the fast time scale, which we call minutes, we follow the cascading process as described above. Then, we complete the model by introducing dynamics for the long time scale. On this time scale, the basic unit of time is a day. In modeling the long-term dynamics, we follow closely the OPA model,<sup>10</sup> which we have used to study the behavior of the power grid.

The evolution goes as follows.

- (1) We introduce an averaged load for the whole system,  $L_A(t) = L_0 \exp(\gamma t)$ , which is a function of time with a constant rate of growth  $\gamma$  which represents the rate of increase in the demand. At the beginning of each day, the loads of the components are uniformly distributed around this mean value within a range  $\Delta L_A$ .
- (2) At the beginning of each day all components are tested for failure, where their probabilities of failure are distributed as in Eq. (1). If there are any failures, a cascade process starts as described above.
- (3) At the end of the day all components that are due for replacement or have failed are replaced.
- (4) After a cascade of a minimum size (greater than 10% of the system size), the maximum load of all components is increased by a factor  $\mu > 1$ . This represents an upgrade of the system in response to failures.

As in the case of the OPA model,<sup>10</sup> this system evolves to a steady state in which failures and replacements are in equilibrium in a time-averaged sense.

### III. BASIC PARAMETERS AND AGENT OPERATION

The operation of the system described in Sec. II is controlled by four parameters. One is the rate of increase in the demand  $\gamma$ , which is taken as a constant. We use  $\gamma = 1.000\ 05$ , which is approximately the average daily rate of increase in the electric power demand in the USA over the past two decades. The second parameter is the load transfer  $p$ . This is a parameter that depends on the nature of the infrastructure. When we have a model of the power transmission grid infrastructure, such as OPA,  $p$  corresponds to the redistribution of power flow. Here we fix its value to be close to but below the critical value  $p_c$ . The motivation comes from analysis of the power systems<sup>14,15</sup> that shows a probability distribution of blackout sizes with a power tail having a decay index that is somewhat larger than the critical value for the CASCADE model.

The two remaining parameters are the upgrade rate of the components  $\mu$  and the replacement time of the components  $\tau_R$ . We use them as the actual control parameters for the operation of the system. There are two independent agents: each controlling one of these parameters and each trying to optimize a utility function involving that parameter. Agent 1 controls the replacement time of the components  $\tau_R$  and agent 2 controls the rate of upgrade. In addition, components are of course also replaced when they fail.

The utility function for both agents has the same structure; it has basically three terms. The first term characterizes the benefit the agent gets from having the system working. This benefit is taken to be proportional to the electricity sold. Following this there are two cost contributions. These are the “real” cost of replacing or upgrading a component and the cost for failure. In principle, the first one is easy to estimate; however the second one is more complicated. It is this term where perception comes in and this must be modeled.

The utility functions for both agents are

$$U_1 = W_p(N - N_F) - W_R N_R - W_{F1} N \Pi(q_1, N_F/N), \quad (3)$$

$$U_2 = W_p(N - N_F) - W_{U1}(\mu - 1) - W_{U2}(\mu - 1)^2 \Gamma - W_{F2} N \Pi(q_2, N_F/N). \quad (4)$$

Here,  $W_p$  is proportional to the price the consumer pays for the “electricity” and is of course the same for both agents. This first term represents the benefit for the agents of having the system working, with increasing benefit for improved operation. On the cost side, for the first agent, the cost is proportional to the number of components replaced,  $W_R$  is the cost of replacing a component, and  $N_R$  is the number of components replaced by maintenance.

For the second agent, the cost of upgrading components has two contributions. First, the second term in Eq. (4), which is independent of time and represents the amortization of upgrade cost over time. It is taken as proportional to the upgrade level  $\mu - 1$  with proportionality constant  $W_{U1}$ . The second contribution, the third term in Eq. (4), is the cost when an upgrade occurs after a cascade, which represents the immediate implementation cost of the upgrade, this term is taken to be proportional to  $(\mu - 1)^2$ ; the constant of proportionality is  $W_{U2}$ . The multiplier  $\Gamma$  is 1 when there is a cascade and 0 otherwise.

Finally, the last term for both functions is the cost (economic, social, and political) to agent  $i$  for having components fail. When no perception of failure is included, this cost is taken to be proportional to the number of failed components that day,  $N_F$ , with a proportionality constant  $W_{Fi}$ . The function  $\Pi$  that multiplies this term reflects the perception of how bad a failure is. Therefore, this is the subjective contribution to the utility function. We model it using functions normally used in prospect theory.<sup>11,12</sup> That is

$$\Pi(q, x) = \frac{x^q}{x^q + (1 - x)^q}. \quad (5)$$

In principle, it is possible that each agent has a different perception of the risk. Different values of  $q$  correspond to different attitudes of the operator regarding risk: risk aversion ( $q < 1$ ), which heavily penalizes the most frequent events, or risk tolerance, which minimizes the cost of the most frequent events ( $q > 1$ ). Examples of these functions for different values of  $q$  are shown in Fig. 2. We also introduce a “dynamic” variation in the model which allows  $q$  to vary with time, making the agents more risk averse immediately after a large blackout followed by a decaying aversion; this variation is described in Sec. V.

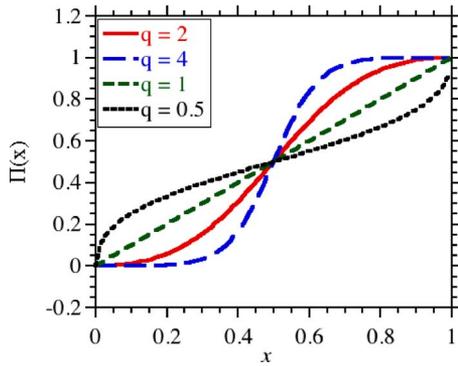


FIG. 2. (Color online) The function  $\Pi$  characterizes the perception of how bad a failure is as a function of the size (which is roughly the inverse of its frequency) for four cases ranging from risk averse  $q=0.5$  to risk taking  $q=4$ .

The operators are designed to make a decision after a given number of days—we have varied the period between 30 and 100 days—regarding the maintenance schedule and the upgrade amount. This choice allows us to average the utility function over a reasonable number of days and avoid its daily high volatility. At the each of each period, each operator makes the decision of what value to use for replacement time and rate of upgrade for the next period, making this decision by examining the following:

- (1) the monthly averaged utility of the last few periods and
- (2) the best averaged utility in the past.

The operator (agents) then tries to optimize this by randomly varying the replacement time and the upgrade increment of the system a fixed amount positively or negatively from its best performance in order to find a possible improvement. Although the agents do not communicate among themselves, they clearly interact implicitly. If one of them changes a basic parameter, the other responds by adapting to the new situation. An example of the evolution of the parameters over time is shown in Fig. 3.

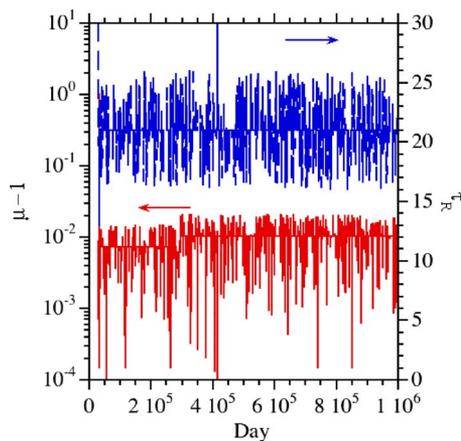


FIG. 3. (Color online) An example of the time evolution of the agent controlled parameters ( $\tau_R$  and  $\mu$ ) for a typical case.

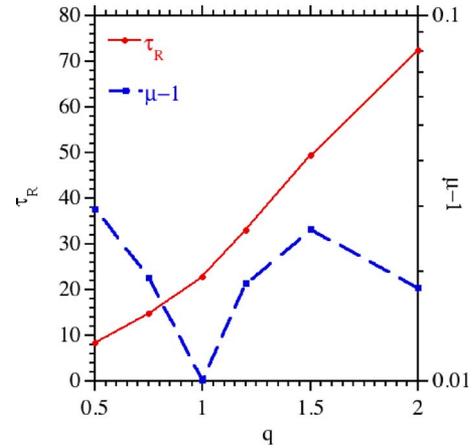


FIG. 4. (Color online) The equilibrium control parameters as a function of  $q$ , the risk comfort parameter, from strongly risk averse ( $q=0.5$ ) to strongly risk taking ( $q=2$ ).

#### IV. NUMERICAL RESULTS OF A SCAN OVER CONSTANT VALUES OF $Q$

We have carried out numerical studies using multiple calculations for each set of parameters. The agents do most of their learning at the beginning of the calculation, and this phase of operation is strongly influenced by the random events occurring at those times. Therefore, longer calculations do not help much in improving statistics; it is better to do multiple calculations. In what follows, we use ten runs for each set of parameters.

We have carried out calculations for several values of  $q$  in the interval  $0.5 < q < 2$  from very risk-averse to strongly risk-taking operation. For each value of  $q$ , the calculations have been carried out for  $10^6$  days.

The first agent, the one controlling the replacement time, reduces the time between replacements as  $q$  decreases and it moves from risk-taking to risk-averse behavior. The replacement time is reduced more than an order of magnitude as the agent moves from operation with  $q=2$  to operation with  $q=0.5$ , as shown in Fig. 4. In going from normal operation  $q=1$  to risk-averse operation  $q=0.5$ , the second agent increases the rate of upgrade. This change in parameters is consistent with the movement toward a risk-averse attitude. It is less clear why we observe the change in optimized behavior of the second agent in going from normal operation to risk-taking operation ( $q=2$ ). There is some increase in the rate of upgrade, which occurs in part to compensate for the increased number of failures due to the large replacement times chosen by the other agent.

The consequences of the agents' choices of operational parameters are very significant. In going from risk-taking operation to risk-averse operation, the frequency of the cascades is strongly reduced, as shown in Fig. 5. This reduction in frequency leads to a reduction in the number of components that fail per unit time, which is also shown in Fig. 5. These results appear to fulfill the goals of a risk-averse operation by minimizing the failures and their occurrences. It should be noted that these improvements actually bear an increased cost; therefore the optimal values of the utility functions are reduced as  $q$  decreases.

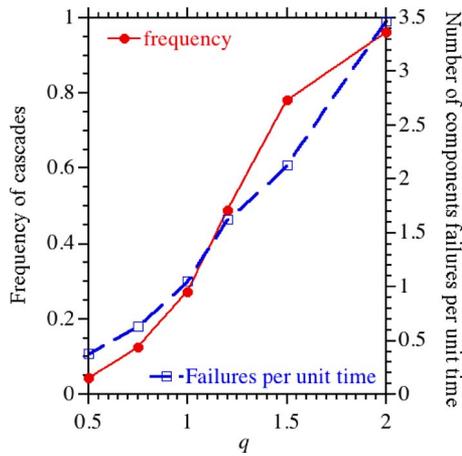


FIG. 5. (Color online) The frequency of failures and number of components failed per unit time both increase as the operation moves from risk averse ( $q=0.5$ ) to risk taking ( $q=2$ ).

Despite the increased cost, these results seem to endorse a risk-averse operation of the system. However, there is a problem. Although the averaged number of failing components is reduced, the probability of an extreme event increases. Here, we characterize an extreme event as an event in which more than half of the components of the system fail. In our calculation the system size is 1000; therefore an extreme event corresponds to 500 failing components.

We can evaluate the probability of an event in which more than 500 components fail for each of the values of  $q$  in the scan. The result is plotted in Fig. 6. We can see a systematic increase in the probability as  $q$  decreases. This change in the probability of an extreme event is a result of the change in the probability distribution function (PDF) of the event size. Once again, we measure the event size by the number of components that fail during a single cascading process.

These changes in the PDF are clearly shown in Fig. 7 where we have plotted the corresponding PDF for three different values of  $q$ : 2, 1, and 0.5. In this figure, we see that the power tail for  $q=2$  has disappeared and the PDF has a different functional form from the other two operational re-

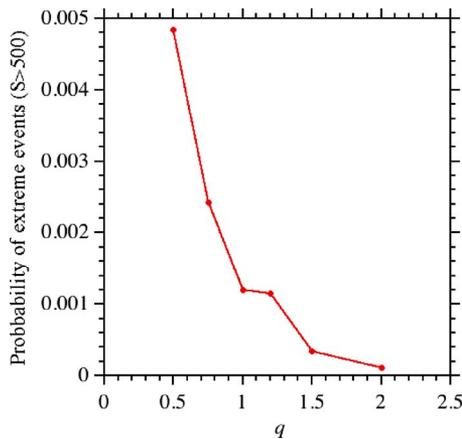


FIG. 6. (Color online) Probability of large events *decreases* as operation moves from risk averse ( $q=0.5$ ) to risk taking ( $q=2$ ).

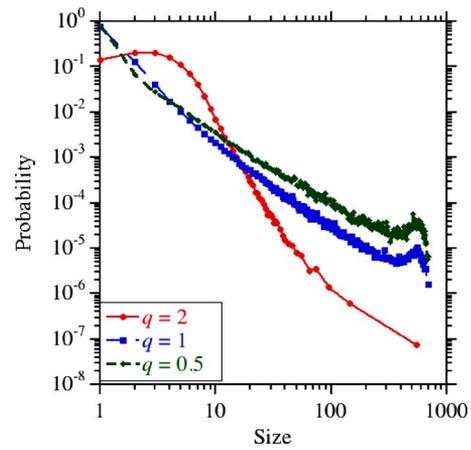


FIG. 7. (Color online) PDF of failure sizes shows heavier tail as operation becomes more risk averse ( $q=0.5$ ).

gimes. In going from normal operation to risk-averse operation the decay index of the power tail goes from 1.8 to 1.45. Clearly, under risk-averse operation, the system appears to be operating in a more critical state.

In moving from risk-taking to risk-averse operation not only the distribution of failures changes but also the dynamics. The clearest manifestation of the change is the change in the long-range time correlations. The risk-taking regime leads to persistence over long time scales while the risk-averse regime leads to antipersistence. The Hurst exponent as operation moves from  $q=2$  to 0.5 changes from 0.6 to 0.3, as shown in Fig. 8. This is consistent with the idea that the operator in a risk-averse regime is trying to avoid all failures. Another way of looking at the effect of risk perception is with a utilization function. Following Kim and Motter<sup>16</sup> we construct such a function, as shown in Fig. 9. Our utilization function is the load over the capacity and because the model does not resolve subday time scales, the utilization we are capturing can be thought of as the peak for the day. We can see only a weak effect of the changing of  $q$ ; however, there does appear to be a peak in the utilization at moderate risk taking values of  $q$  (around  $q=1.5$ ). This would suggest once

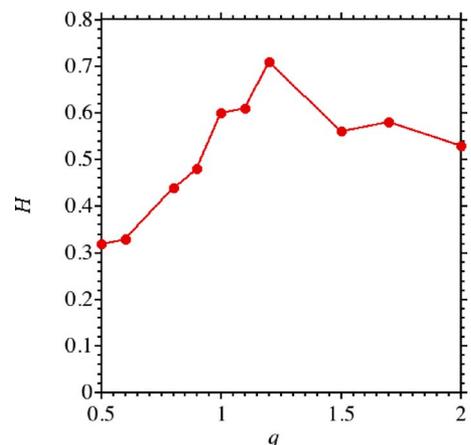


FIG. 8. (Color online) The Hurst exponent ( $H$ ) goes from persistent ( $H>0.5$ ) to antipersistent ( $H<0.5$ ) as operation is moved from risk taking ( $q=2$ ) through normal to risk averse ( $q=0.5$ ).

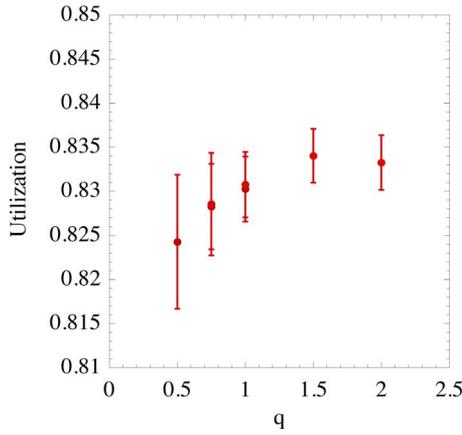


FIG. 9. (Color online) The utilization appears to have a maximum with risk taking behavior. Although the uncertainties make any strong statement impossible and the actual value does not change much, there is an apparent maximum near  $q=1.5$ .

again that risk aversion is not the most effect operational tactic. We should note that the actual values for the utilization should not be compared to those of Kim and Motter because of the daily peak limitation we mentioned previously.

### V. RISK-AVERSE OPERATION AS A REACTION TO AN EXTREME EVENT

Another possible way in which risk aversion may affect the operation of infrastructure systems is when a risk-averse policy is the reaction to some extreme event. In what follows, we are going to incorporate such possibility into our generalized cascade model by making the exponent  $q$  as a function of time.

We will use the following formulation of the problem. We start with  $q=1.0$ . If an event of size  $S$ , greater than a given threshold  $S_{th}$ , occurs at time  $t_0$ , then  $q$  falls to  $q_0=0.5$ , and then evolves in the following way:

$$q(t) = 1 + (q_0 - 1)\exp(-(t - t_0)/T_0). \tag{6}$$

We choose  $q_0=0.5$  because we want a risk-averse reaction to the event. The parameter  $T_0$  is the time after which the system (agent) has “forgotten” the event. In the present calculations we take  $T_0=500$  days. Later, we will discuss the effect of changing this characteristic memory decay time.

We have carried out a scan over possible values of the event threshold value  $S_{th}$ . As before, for each threshold value, we do ten calculations in order to accumulate good statistics over the various events and agent behavior responses to those events.

The choice of the operational parameter for each different threshold value is consistent with the risk-averse attitude of the agents, as we have seen in Sec. IV. This is shown in Fig. 10, where we plot the average value of  $\mu$  and  $\tau_R$  as a function of the event threshold. Here, the averages are taken over the full length of the simulations, that is  $10^6$  days, and over the ten different simulations for each set of parameters.

As the threshold for the event triggering the risk-averse reaction decreases, the system operates in a risk-averse mode

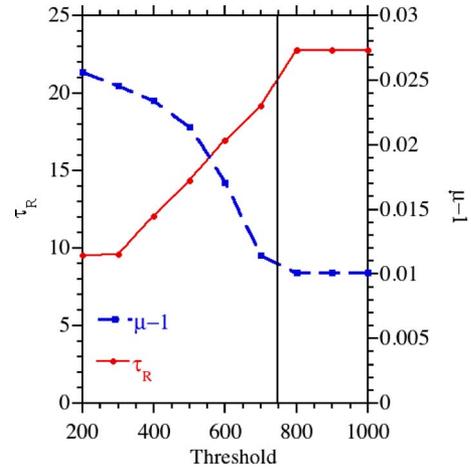


FIG. 10. (Color online) The average value of  $\mu$  and  $\tau_R$  as a function of the “learning” event threshold for the system that forgets about events after a given decay time.

of operation for a longer time. Therefore decreasing the threshold lengthens risk-averse operation relative to normal operation. As the risk-averse operation gets longer, we see a reduction in the replacement time and an increase in the rate of upgrade, as shown in Fig. 4.

Note that when the threshold is above 750 there is no effective threshold at all because there are no events greater than this size. Therefore all such thresholds are equivalent and they serve as the reference case for normal operation.

What are the consequences of the change in behavior of the agents? As they operate in a risk-averse mode for longer periods of time, corresponding to lower thresholds, there is a clear reduction in the frequency of the cascades, as we have seen in Sec. IV. Figure 11 illustrates that point: as the value of the threshold is reduced, the frequency decreases.

The situation is somewhat different with regard to the number of failures per unit time, which we have also plotted in Fig. 11. We can see that for large thresholds, for which only a few events trigger the risk-averse attitude, there is a decrease in failures per unit time because of the decrease in

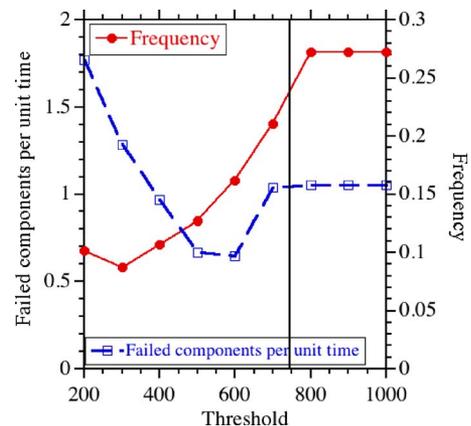


FIG. 11. (Color online) Frequency of failures and number of failed components and a function of the threshold for risk-averse behavior. This plot suggests there may be an optimal threshold for such behavior which minimizes both the frequency and size of the failures.

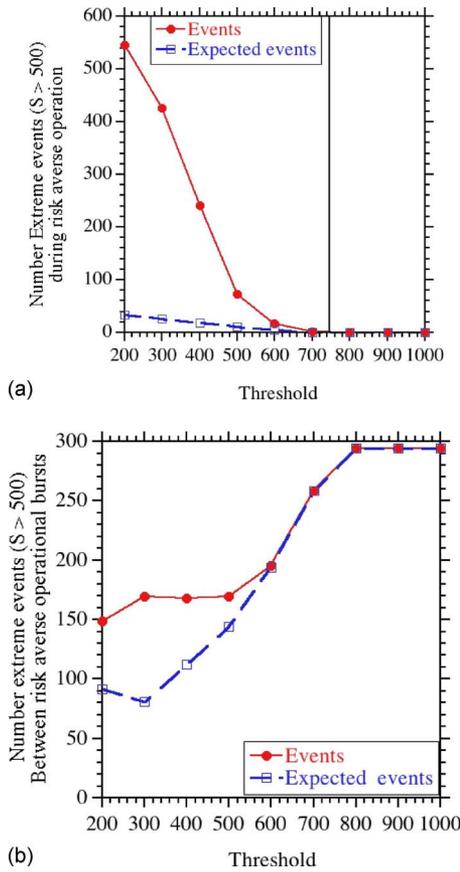


FIG. 12. (Color online) Number of extreme events (larger than 500) (a) during risk-averse operation (top) and (b) between risk-averse operation (bottom) compared to expected events if there is no risk-averse operation.

frequency of the events. However, as the risk-averse attitude is triggered more often, that is, for smaller thresholds, the increase in the average size of the events compensates for the decrease in frequency and eventually the number of failures per unit time is higher than in the reference case. As we saw in Sec. IV, risk-averse operation increases the probability of extreme events and this causes an increase in the number of failures if a risk-averse attitude is triggered too often. This suggests that there might be an optimal threshold for the onset of risk-averse operation.

We want to examine the effect of risk-averse operation on extreme events in more detail. To do so, we analyze separately the period of time during which the agents are risk averse, distinct from the period when they are not risk averse. Using the reference case, we can calculate the probability of an event larger than 500. From this probability, we calculate the expected number of extreme events in the two phases of operation for the different thresholds. These results are compared to the real number of extreme events obtained with those calculations. The results are plotted and compared in Figs. 12(a) and 12(b). We can see that in general risk-averse operation leads to a higher number of extreme events during both phases, the risk-averse phase and the one following afterward. From these figures, we see that the increase is clearly more dramatic during the risk-averse phase.

Risk-averse operation affects the PDF of the cascade sizes, as we have seen in Sec. IV. In Sec. IV, we have shown

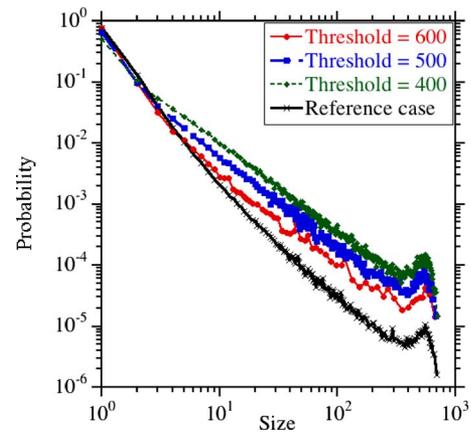


FIG. 13. (Color online) PDFs of threshold of 600, 500, and 400 and reference cases showing a heavier tail with the thresholding.

the PDF of the event sizes for the reference case,  $q=1$ . We have used a sample of  $10^6$  cascades in the calculation of the PDF. Here, we examine the same quantity selecting cases with thresholds 600, 500, and 400. For each case, we have combined the data of ten different calculations during the periods of risk-averse operation. This gives samples of 34 000, 80 000, and 134 000 points, respectively. In Fig. 13, we show these PDFs for these cases.

We can see that all PDFs have a region governed by a power law, but each clearly has a different decay index. In the reference case, the PDF falls with an exponent  $-1.8$ , while for risk-averse operation the decay index is about  $-1.4$  for the three cases plotted. The three risk-averse cases have the same slope; the only difference is in the value at which the algebraic tail region begins. As the threshold decreases this value also decreases and the region of the power tail widens. The change in the PDF is clear. This result again shows that risk-averse operation brings the system to a more critical operating point.

The PDF for operation with a constant  $q=0.5$  is identical to the PDF calculated during the risk-averse operation for the case of threshold 600, see Fig. 14. The case with threshold 600 is a case with very few regions of risk-averse operation

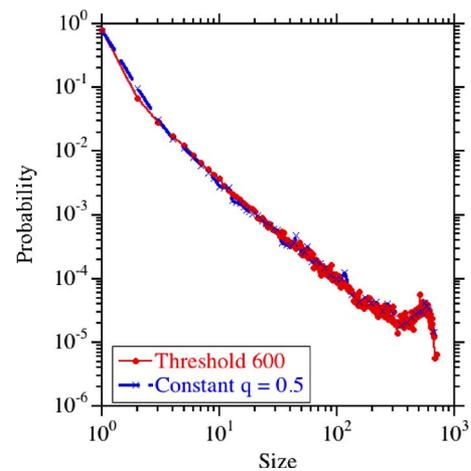


FIG. 14. (Color online) PDFs of  $q=0.5$  and threshold of 600 cases, which are virtually identical.

and clearly these regions did not overlap. This result shows the consistency between the constant  $q=0.5$  operation and intermittent operation in response to large events.

The results presented in this section depend on the length of risk-averse operation. We have done a set of calculations for different values of the decay parameter  $T_0$ , which is a measure of how long the risk-averse reaction to an extreme event lasts. These calculations have revealed two different regimes. For small values of the decay time,  $T_0 < 500$  days, the time is too short for the agents to carry out sufficient learning and adapt to the new situation, and the results are somewhat erratic. For large values of the decay time,  $T_0 > 500$  days, the response is consistent with the previous results. An increase in the decay time has the same effects as a decrease in the threshold value because it represents an increase in risk-averse operation.

## VI. CONCLUSIONS

In this paper, we presented a simple model of the operation of an infrastructure system. This model is a generalization of the CASCADE model, which has been studied in detail in the past. This generalization transforms the CASCADE model from a probabilistic model to a dynamic model.

The dynamic evolution of the system over a long time scale is governed by a daily increase in consumer demand that raises the overall load on the system and an engineering response to failures that involves upgrading of the components. The system is controlled by adjusting two parameters: the upgrading rate of the components  $\mu$  and the replacement time of the components  $\tau_R$ . Two agents operate the system by each selecting one of those parameters.

The utility functions used by the agents to optimize performance incorporate some perceptions of the events that affect the decision making of the agents. The agents are characterized by three aspects of their (social) behavior:

- (1) their risk-averse and risk-taking attitudes while operating the system;
- (2) their response to large events, which can trigger a change in their behavior; and
- (3) the effect of learning time on adapting to new conditions.

These three agent behaviors affect the performance of the infrastructure system.

In going from risk-taking to risk-averse operation there is a reduction in the frequency of failures and in the number of failures per unit time. However, risk aversion brings an increase in the probability of extreme events. During risk-averse operation, the PDF falls off with a smaller exponent than that found in normal operation. This is in general a very unwelcome change because large events are of much higher cost.

When risk-averse operation is triggered in response to extreme events, we obtain similar results as we find in the case of continuous risk-averse operation, but the probability of extreme events can be even higher than in the continuous operation if this reaction is triggered too often, that is if the threshold for entering risk-averse operation is relatively low.

Finally, one of the parameters that seem to be most important in determining the effectiveness of the agents is the learning time they are allocated after their behavior changes. If the time to react to changing conditions is too short, the agents have difficulties finding optimal operational conditions.

## ACKNOWLEDGMENTS

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